

Low temperature $1/f$ noise in microwave dielectric constant of amorphous dielectrics in Josephson qubits

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The accurate analytical solution for the low temperature $1/f$ noise in a microwave dielectric constant of amorphous films containing tunneling two-level systems (TLSs) is derived within the standard tunneling model including the weak dipolar or elastic TLS-TLS interactions. The results are consistent with the recent experimental investigations of $1/f$ noise in Josephson junction qubits including the power law increase of the noise amplitude with decreasing temperature at low temperatures $T < 0.1\text{K}$. The long time correlations needed for $1/f$ noise are provided by the logarithmic broadening of TLS absorption resonances with time due to their interaction with neighboring TLSs. The noise behavior at higher temperatures $T > 0.1\text{K}$ and its possible sensitivity to quasi-particle excitations are discussed.

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$1/f$ noise exists in a variety of physical systems,¹⁻⁴ and it dramatically restricts performance of modern electronic and quantum nanodevices.⁵⁻⁷ The inverse frequency dependence of the noise power spectral density, $S_{xx}(f) \propto 1/f$, is a consequence of a logarithmically slow relaxation. The slow dynamics is often associated with the random ensemble of fluctuators possessing a logarithmically uniform spectrum of relaxation times.^{2,3,8-10} Such fluctuators do exist in amorphous solids in the form of universal tunneling two-level systems (TLSs).¹¹ With the advent of superconducting quantum bits (qubits) based on Josephson junctions,^{7,12} a comprehensive study of the noise properties due to TLSs has become crucial for the achievement of high-fidelity quantum computation. TLSs are ubiquitous, appearing in wiring dielectrics, Josephson junction barriers and other disordered insulating regions. The deleterious effects of the coupling of TLSs to the qubit are associated with the resonant absorption of the qubit energy by TLSs in the microwave frequency range, as well as by $1/f$ noise in the microwave dielectric constant,⁷ resulting in qubit decoherence.

Dissipation and $1/f$ noise in superconducting resonators due to TLSs have been extensively studied.¹³⁻¹⁷ Recently, it was found that the $1/f$ noise amplitude in high quality superconducting resonators increases with decreasing temperature as $T^{-1-\eta}$, with $\eta \approx 0.3$.¹⁶ This dependence was considered as being incompatible with the standard tunneling model (STM).^{16,18}

A qualitative theoretical model interpreting this recent experiment has been proposed in Refs. 16 and 18, suggesting an energy-dependent TLSs density of states (DOS), $g(E) \approx E^\eta$, in contrast with the standard TLS model suggesting a constant DOS.¹¹ The authors used their assumption to interpret the additional exponent η in the noise temperature dependence and the anomalous temperature dependence of TLSs decoherence rate dis-

covered earlier using a single TLS spectroscopy.¹⁹ This assumption conflicts with earlier experimental data showing energy independence of the DOS, leading to logarithmic temperature dependence of the dielectric constant and sound velocity^{20,21} (see however Ref. 22).

In this paper we derive an exact analytical solution for the low temperature $1/f$ noise in a microwave dielectric constant of amorphous films, assuming a homogeneous DOS as in the STM. This solution can be directly compared with the available experimental data and used to quantitatively characterize TLSs properties which is important for understanding of the nature of TLSs and the reduction of their destructive effects in superconducting quantum devices. Particularly, we show that by properly considering the logarithmic temperature dependence of the noise power spectral density, the $T^{-1-\eta}$ dependence of the amplitude of $1/f$ noise on temperature can be explained within the assumptions of the STM and predict the noise dependence on the external field amplitude. The derivation focuses on the relevant regime of low temperatures, where the thermal energy is much smaller than the field quantization energy, namely $k_B T \ll \hbar\omega$.

At higher temperature, $k_B T \geq \hbar\omega$, we argue that TLSs based models cannot explain the noise temperature dependence. An alternative noise source in this regime is discussed by employing the possible contribution of quasi-particles excitations in the superconductors. It is demonstrated that such excitations may be responsible for the anomalous temperature dependence of the relaxation and decoherence rates of TLSs, observed in this regime.¹⁹

Consider the TLSs contribution to the dielectric constant measured using the input field, F_{AC} , applied along the z axis at a microwave frequency ω and low temperature $k_B T \ll \hbar\omega$. In this regime the resonant contribution of TLSs to the real part of the dielectric constant domi-

nates and can be expressed in the form^{18,20,23,24}

$$\frac{\epsilon_{TLS}(t)}{\epsilon} = \frac{4\pi}{V\epsilon} \sum_i \tanh\left(\frac{E_i}{2k_B T}\right) \times \frac{(E_i(t) - \hbar\omega)p_{tr,iz}^2}{(E_i(t) - \hbar\omega)^2 + T_{2i}^{-2}(1 + p_{tr,iz}^2 F_{AC}^2 T_{1i} T_{2i} / \hbar^2)}. \quad (1)$$

Here, V and ϵ represent the sample volume and bare dielectric constant. The summation is taken over all TLSs i having transition dipole moments $\mathbf{p}_{tr,i}$, relaxation and decoherence times T_{1i} and T_{2i} , respectively, and time-dependent energies $E_i(t)$. The z component of the transition dipole moment is related to the corresponding component of the TLS dipole moment, p_{iz} , by $p_{tr,iz} = p_{iz}\Delta_{0i}/E_i$,^{20,23} where Δ_{0i} is the tunneling amplitude of TLS i . Following the STM,^{11,20} we assume that TLSs possess the universal distribution $P(\Delta_0, E) = P_0 E / (\Delta_0 \sqrt{E^2 - \Delta_0^2})$ with respect to their energies and tunneling amplitudes, where P_0 is a material dependent constant.

The time dependence of the energies is induced by the spectral diffusion caused by weak dipolar or elastic TLS-TLS interaction.²⁵ The time dependence of the energy can be treated classically because at the temperatures under consideration, $k_B T \ll \hbar\omega \approx E$, the equilibrium thermal fluctuations can induce transitions of TLSs only with an exponentially small probability $e^{-E/k_B T}$.

Relaxation of TLSs is caused by phonon emission or absorption, with the relaxation rate^{20,25,26}

$$\frac{1}{T_{1i}} = A \left(\frac{\Delta_{0i}}{E_i}\right)^2 \frac{E_i^3}{k_B^3} \coth\left(\frac{E_i}{2k_B T}\right), \quad (2)$$

where the proportionality constant $A \sim 10^8 \text{s}^{-1} \text{K}^{-3}$ characterizes the TLS-phonon interaction.²⁶ The TLS deco-

herence rate is composed of the contributions of relaxation and pure phase decoherence, $T_{2i}^{-1} = (2T_{1i})^{-1} + T_{\varphi,i}^{-1}$, where the phase decoherence rate $T_{\varphi,i}^{-1}$ is determined by the TLS spectral diffusion induced by its interaction with neighboring thermal TLSs, i.e. TLSs for which $E \approx \Delta_0 \approx k_B T$.²⁵ This rate is given by^{25,26}

$$\frac{1}{T_{\varphi,i}} = \sqrt{40 \frac{|\Delta_i|}{E_i} \frac{\chi k_B T \cdot AT^3}{\hbar}}, \quad (3)$$

where $\Delta_i = \sqrt{E_i^2 - \Delta_{0i}^2}$ is the TLS asymmetry and the dimensionless constant $\chi = P_0 U_0 \sim 5 \cdot 10^{-4}$ represents the universal product of the TLSs DOS and their $1/R^3$ interaction strength.²⁷ The product $\chi k_B T$ represents the typical interaction with thermal TLSs and AT^3 represents the relaxation rate of thermal TLSs. Although the use of Eq. (3) in the expression for the dielectric constant [Eq. (1)] has not been justified theoretically, its approximate relevance was demonstrated experimentally.²⁴ One should notice that in the linear regime of weak external field the related term is negligible in the noise spectral density (see Eq. (9) below and estimates of parameters of this equation in the next two paragraphs). In the opposite regime of a very strong field the accuracy of our results is limited to the above assumption and may need a special consideration.

The noise is determined by the time-dependent correlation function of the dielectric constant fluctuations $S_{xx}(t) = \langle \delta\epsilon_{TLS}(t) \delta\epsilon_{TLS}(0) \rangle / \epsilon^2$, where $\delta\epsilon_{TLS}(t) \equiv \epsilon_{TLS}(t) - \langle \epsilon_{TLS} \rangle$. Correlations between different resonant TLSs contributing to the noise can be neglected because of the weakness of the TLS-TLS interactions (see e.g., Ref. 27). The correlation function can then be expressed as

$$S_{xx}(t) = \frac{(4\pi)^2}{V^2 \epsilon^2} \sum_i p_{iz}^4 \tanh^2\left(\frac{E_i}{2k_B T}\right) \times \left\langle \frac{E_i(t) - \hbar\omega}{(E_i(t) - \hbar\omega)^2 + T_{2i}^{-2}(1 + p_{tr,iz}^2 F_{AC}^2 T_{1i} T_{2i} / \hbar^2)} \cdot \frac{E_i(0) - \hbar\omega}{(E_i(0) - \hbar\omega)^2 + T_{2i}^{-2}(1 + p_{tr,iz}^2 F_{AC}^2 T_{1i} T_{2i} / \hbar^2)} \right\rangle. \quad (4)$$

The correlation function in Eq. (4) can be evaluated by means of the distribution function $D(E|E_0, t)$ for the TLS energy E at time t , assuming $E(0) = E_0$ at $t = 0$.²⁵ For $1/R^3$ interaction, this distribution function is a Lorentzian

$$D(E|E_0, t) = \frac{1}{\pi} \frac{W(t)}{W^2(t) + (E - E_0)^2}, \quad (5)$$

with a characteristic width $W(t) = W_0(t)|\Delta|/E$, where

$$W_0(t) = \frac{\pi^2 \chi k_B T}{3\hbar} \int_0^\infty \frac{dy}{\cosh^2 y} \int_0^1 \frac{1 - e^{-rt/T_1(y)}}{r} dr, \quad (6)$$

and $T_1^{-1}(y) = 8AT^3 y^3 \coth y$.

Low frequency $1/f$ noise is determined by long times $t \geq 1 \text{s} \gg (AT^3)^{-1}$,¹⁶ where $(AT^3)^{-1}$ estimates the minimum relaxation time of thermal TLS. In this limit the width of the distribution $W_0(t)$ grows logarithmically

with time and can be approximated as

$$W_0(t) \approx \frac{\pi^2}{3\hbar} \chi k_B T \ln(3.3 \cdot AT^3 t). \quad (7)$$

This logarithmic increase of the resonance width is responsible both for the appearance of $1/f$ noise and for the additional exponent η in the temperature dependence $T^{-1-\eta}$ of the noise amplitude. This dependence has not been considered in Ref. 18, leading the authors to the

conclusion that the experimental results are inconsistent with the STM.

At low temperatures $W(t) \ll \hbar\omega$ and one can perform averaging of the correlation function in Eq. (4) over the distribution of energy fluctuations [Eq. (5)] and over initial TLS energies within the resonant approximation. Straightforward evaluation of integrals with respect to $E(t)$ and $E(0)$ yields

$$S_{xx}(t) = \tanh^2\left(\frac{\hbar\omega}{2k_B T}\right) \frac{4\pi^3 P_0}{\hbar V \epsilon^2} \int_0^1 \frac{dx}{x(1-x^2)} \int_0^1 dy \left\langle \frac{p_0^4 x^4 y^4}{3W_0(t) + \frac{T_2^{-1}(x)}{\sqrt{1-x^2}} \sqrt{1 + (xyp_0 F_{AC})^2 T_1(x) T_2(x)/\hbar^2}} \right\rangle, \quad (8)$$

where $x = \Delta_0/\hbar\omega$ and $y = \cos\theta$. Averaging in Eq. (8) is made over absolute values of TLS dipole moment, p_0 , forming an angle θ with the AC field. In the numerical calculations below we assume $p_0 \sim 5D$, which is well justified experimentally.⁷ The relaxation rate $T_1^{-1}(x)$ is given by Eq. (2) with E_i replaced by $\hbar\omega$ and Δ_{0i}/E_i replaced by x . Similarly, $T_2^{-1}(x) = (2T_1(x))^{-1} + T_\varphi^{-1}(x)$, where $T_\varphi^{-1}(x)$ is given by Eq. (3) with $\Delta_i/E_i = \sqrt{1 - (\Delta_{0i}/E_i)^2}$ replaced by $\sqrt{1 - x^2}$.

The power spectral density of noise, $S_{xx}(f)$, can be evaluated as a Fourier transform of $S_{xx}(t)$ [Eq. (8)] in the low frequency limit $fT_1, f/W \ll 1$. It has the pure $1/f$ spectrum if the function $S_{xx}(t)$ depends on time as $A - B \ln|t|$, which has a Fourier transform $B/(2f)$ at

$f \neq 0$. The correlation function $S_{xx}(t)$ can be represented using the expansion near $t_f \approx 1/(2\pi f)$ in the approximate form $S_{xx}(t) \approx S_{xx}(t_f) + \frac{dS_{xx}(t_f)}{d \ln t_f} \ln(t/t_f)$ (higher order expansion terms are smaller by the factor $\ln^{-1}(AT^3/f) \approx 0.1$ for the small frequency of interest $f \approx 0.1\text{Hz}$). The $1/f$ noise power spectral density can then be expressed as $S_{xx}(f) = -(1/2f) dS_{xx}(t_f)/d \ln t_f$.

For a quantitative comparison of the theory with the experiment it is convenient to introduce a volume independent parameter in a similar way to the experimentally determined Hooge's constant for $1/f$ conductivity noise in semiconductors.²⁸ We define this parameter as

$$\alpha_{TLS} \equiv \frac{P_0 V k_B T f S_{xx}(f)}{\tan^2 \delta} = \frac{9\pi}{8\langle p_0^2 \rangle^2 \hbar^2} \int_0^1 \frac{dx}{x(1-x^2)} \int_0^1 dy \left\langle \frac{p_0^4 x^4 y^4 \chi(k_B T)^2}{\left[3W_0(1/(2\pi f)) + \frac{T_2^{-1}(x)}{\sqrt{1-x^2}} \sqrt{1 + (xyp_0 F_{AC})^2 T_1(x) T_2(x)/\hbar^2}\right]^2} \right\rangle, \quad (9)$$

which is the ratio of the noise spectral density multiplied by the number of thermal TLSs, $N_T = P_0 V k_B T$, to the squared average loss tangent due to the TLSs, $\tan \delta = \langle \epsilon'' \rangle / \epsilon = (4\pi^2/3\epsilon) P_0 \langle p_0^2 \rangle \tanh(\hbar\omega/2k_B T)$. One should notice the logarithmic weak frequency dependence of the noise amplitude equivalent to a hardly distinguishable $f^{0.1}$ dependence for the typical system parameters (see the caption to Fig. 1).

Consider the noise amplitude temperature dependence in the linear regime $F_{AC} \rightarrow 0$. For typical TLS parameters $\chi \approx 5 \cdot 10^{-4}$ and $A \approx 10^8 \text{s}^{-1} \text{K}^{-3}$,²⁶ one has $W_0(T) \sim 10^8 T (\text{s}^{-1})$, $T_2^{-1} \approx 10^9 T^2 (\text{s}^{-1})$ and $T_1^{-1} \approx 10^6 \text{s}^{-1}$. Consequently, the second term in the denominator of the integrand in Eq. (9) can be approximately neglected at temperatures $0.01 \text{K} < T < 0.1 \text{K}$. The tem-

perature dependence of the noise amplitude can thus be approximated by $S_{xx}(f) \propto T^{-1} \ln^{-2}(3.3 \cdot AT^3/2\pi f)$. The approximate exponent in the power law temperature dependence $S_{xx}(f) \propto T^{-1-\eta}$ observed in the experiment can be estimated as $\eta = -1 - d \ln S_{xx}(f)/d \ln T = -d \ln \alpha_{TLS}/d \ln T = 6/\ln(3.3 \cdot AT^3/2\pi f)$.⁸ Setting $f = 0.1 \text{Hz}$,¹⁶ we obtain $\eta \approx 0.42$, in agreement with the experimental observation. The actual dependence is slightly weaker because of the logarithmic integral in Eq. (9). Thus, the reported anomalous temperature dependence of the $1/f$ noise amplitude in superconducting resonators at low temperatures can be interpreted within the framework of the standard tunneling model.

The temperature dependence of the noise parameter

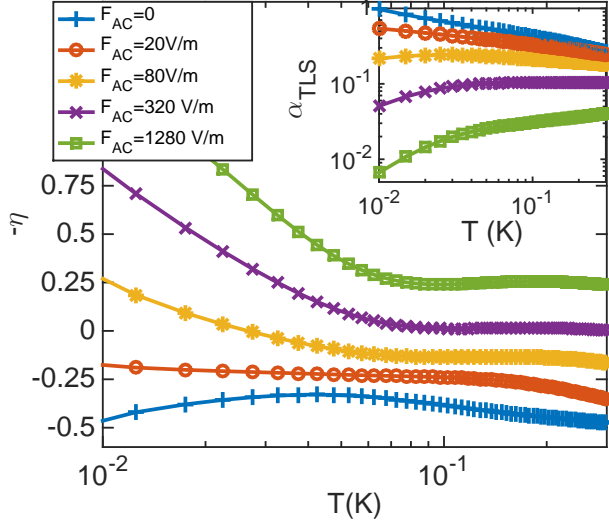


FIG. 1: The exponent η in the noise temperature dependence $\alpha_{TLS} \propto T^{-\eta}$ and the noise parameter α_{TLS} (inset) vs. temperature. TLS parameters are chosen as²⁶ $p_0 = 5D$, $P_0 U_{rms} = 5 \cdot 10^{-4}$, $A = 10^8 \text{s}^{-1} \text{K}^{-3}$, $\omega = 2\pi \cdot 5 \text{GHz}$.

α_{TLS} [Eq. (9)] and the exponent η extracted from it are shown in Fig. 1 for various external field amplitudes. It is evident that in the linear regime, $F_{AC} \rightarrow 0$, the temperature dependence observed in the experiment in the regime $k_B T \ll \hbar\omega$ is qualitatively reproduced. The accurate comparison of the theory with the experiment is the subject of a separate work that will help to determine the system parameters from the optimum data fit.

It should be emphasized that our results do not rule out the possibility of some energy dependence of the DOS. For the specific case of $1/f$ noise it shows that a significant contribution to the exponent η may arise due to the broad spectrum of relaxation times in amorphous systems, even if the DOS is energy independent. However, other deviations from the STM, such as the non-integer exponents of the specific heat and thermal conductivity may originate from an energy dependence of the DOS at low energies. Further investigation is thus necessary in order to shed light on the energy dependence of the DOS. This can be achieved by additional measurements, such as temperature and frequency dependent dielectric losses or internal friction in the plateau regime, which should be both sensitive to the energy dependence of the DOS.²⁰

The strong reduction of the noise with increasing field amplitude, as well as the weakening of its temperature dependence are clearly seen in Fig. 1. These findings are in agreement with the experimental results.¹⁶ Yet, in the large field limit, $F_{AC} > (\hbar W_0(t)/p_0) \sqrt{T_2/T_1} \approx 1000 \text{V/m}$, Eq. (9) predicts $S_{xx} \propto T F_{AC}^{-2}$, while the observed field dependence is weaker.¹⁶ This might be due to nonequilibrium heating⁵ and insufficiently large AC field used experimentally. A detailed analysis requires understanding of the mechanisms of heat exchange between

the superconducting circuit and the environment, which is beyond the scope of this letter.

At high temperatures, $k_B T > \hbar\omega$, Eq. (9) as well as the earlier work^{16,18} predict the strong reduction of the noise with increasing temperature due to the factor $\tanh^2(E_i/2k_B T) \propto T^{-2}$. Such temperature dependence does not fit the experimental observation,¹⁶ which reports a weaker temperature dependence. We expect that different mechanism may be responsible for the $1/f$ noise in this regime. According to our preliminary analysis this cannot be the relaxational interaction of TLS with the AC field²⁰ since it does not lead to $1/f$ noise. Indeed, the low frequency noise should be due to slow TLSs with the large relaxation times $T_1 \approx 1/2\pi f$. They can indeed contribute to the fluctuations of a zero frequency dielectric constant¹⁰ while their response to an AC field vanishes as f/ω .²⁰

Alternatively, the noise in dielectric tunneling barrier incorporated in superconducting devices may be due to quasi-particle excitations. In particular, a recent study has revealed an anomalous temperature dependence of the relaxation and decoherence rates of single TLSs in the amorphous tunnel barrier of a superconducting phase qubit, showing a deviation from the single-phonon relaxation rate [Eq. (2)] at temperatures above 0.1K.¹⁹ As shown in Fig. 2, the relaxation rate of these TLSs can be fitted by combining the contributions to the relaxation due to TLS-phonon interaction and the thermally activated quasi-particle contribution²⁹ as $T_1^{-1}(T) = A \coth(E/2k_B T) + B e^{-\Delta/2k_B T}$. For both TLSs studied in Ref. 19, the fitting parameter Δ is close to the Al superconducting gap $3.95 k_B \text{K}$. These quasi-particles may also be responsible for $1/f$ noise at temperatures exceeding 0.1K.

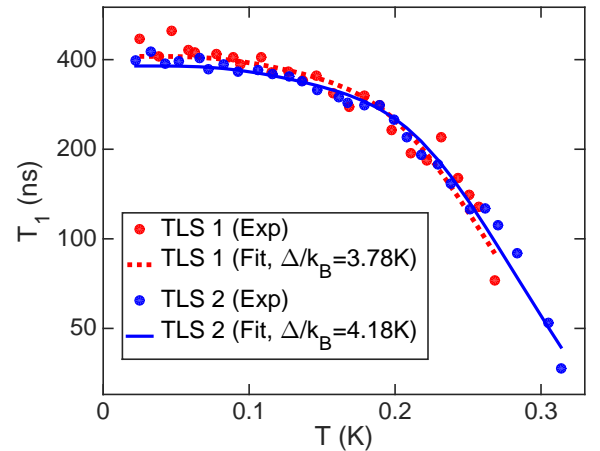


FIG. 2: Experimental data¹⁹ for temperature dependent relaxation times of two different TLSs interpreted assuming the presence of quasi-particles.

In conclusion, we calculated the $1/f$ noise in microwave dielectric constant produced by TLSs in amorphous solids. The STM involving the long range TLS-

TLS interactions has been used. The results are consistent with the recent experimental data at low temperature, whereas at higher temperature other mechanism may be responsible for the noise, possibly one associated

with broken Cooper pairs.

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- ¹ P. Dutta and P. M. Horn, Rev. Mod. Phys. **53**, 497 (1981).
 - ² M. B. Weissman, Rev. Mod. Phys. **60**, 537 (1988).
 - ³ Sh. M. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, 1996).
 - ⁴ W. Schottky, Ann. Phys. (Berlin) **362**, 541 (1918).
 - ⁵ S.-I. Han *et al.*, in *EUV, X-ray, and Gamma-Ray Instrumentation for Astronomy IX*, O. H. Seigmund and M. A. Gummin, Editors, Proc. of SPIE **3445**, 660 (1998); D. McCammon, in *Cryogenic Particle Detection* edited by C. Enss, Top. Appl. Phys. **99**, 35 (Springer, Berlin, 2005).
 - ⁶ D. McCammon *et al.*, Phys. Stat. Sol. b **230**, 197 (2002).
 - ⁷ J. M. Martinis *et al.*, Phys. Rev. Lett. **95**, 210503 (2005).
 - ⁸ A. L. Burin, B. I. Shklovskii, V. I. Kozub, Y. M. Galperin, and V. Vinokur, Phys. Rev. B **74**, 075205 (2006).
 - ⁹ Y. Lemonik and I. Aleiner, Phys. Rev. B **90**, 184203 (2014).
 - ¹⁰ C. C. Yu, J. Low Temp. Phys. **137**, 251 (2004).
 - ¹¹ P. W. Anderson, B. I. Halperin, and C. M. Varma, Philos. Mag. **25**, 1 (1972); W. A. Phillips, J. Low-Temp. Phys. **7**, 351 (1972).
 - ¹² J. Q. You and F. Nori, Phys. Today **58**, 42 (2005); J. Clarke and F. K. Wilhelm, Nature **453**, 1031 (2008).
 - ¹³ C. Neill *et al.*, Appl. Phys. Lett. **103**, 072601 (2013).
 - ¹⁴ K. W. Murch, S. J. Weber, E. M. Levenson-Falk, R. Vijay, and I. Siddiqi, Appl. Phys. Lett. **100**, 142601 (2012).
 - ¹⁵ J. Burnett, T. Lindström, M. Oxborrow, Y. Harada, Y. Sekine, P. Meeson, and A. Ya. Tzalenchuk, Phys. Rev. B **87**, 140501(R) (2013).
 - ¹⁶ J. Burnett *et al.*, Nat. Commun. **5**, 4119 (2014).
 - ¹⁷ K. D. Osborn and coworkers, unpublished, in preparation.
 - ¹⁸ L. Faoro and L. B. Ioffe, Phys. Rev. B **91**, 014201 (2015).
 - ¹⁹ J. Lisenfeld, C. Müller, J. H. Cole, P. Bushev, A. Lukashenko, A. Shnirman, and A. V. Ustinov, Phys. Rev. Lett. **105**, 230504 (2010).
 - ²⁰ S. Hunklinger and A. K. Raychaudhary, Progr. Low Temp. Phys. **9**, 267 (1986).
 - ²¹ M. von Hauneder, U. Strom, and S. Hunklinger, Phys. Rev. Lett. **44**, 84 (1980).
 - ²² In a recent work [S. T. Skacel *et al.*, Appl. Phys. Lett. **106**, 022603 (2015)], an anomalously strong energy dependence of the DOS has been reported in *a*-SiO film. This dependence looks much stronger than the one assumed in Ref. 18 and is probably related to the lack of oxygen compared to the standard SiO₂ stoichiometry. The material can have a large number of dangling bonds possibly responsible for a "non-glassy" behavior.
 - ²³ H. M. Carruzzo, E. R. Grannan, and C. C. Yu, Phys. Rev. B **50**, 6685 (1994).
 - ²⁴ M. Von Schickfus and S. Hunklinger, Phys. Lett. A **64**, 144, (1977).
 - ²⁵ J. L. Black and B. I. Halperin, Phys. Rev. B **16**, 2879 (1977).
 - ²⁶ A. L. Burin, J. M. Leveritt, G. Ruyters, C. Schötz, M. Bazrafshan, P. Fassel, M. von Schickfus, A. Fleischmann, and C. Enss, Euro Phys. Lett. **104**, 57006 (2013).
 - ²⁷ C. C. Yu and A. J. Leggett, Comments Condens Matter Phys **14**, 231 (1988).
 - ²⁸ F. N. Hooge, Physica **60**, 130 (1972); F. N. Hooge, T. G. M. Kleinpenning, and L. K. J. Vandamme, Rep. Prog. Phys. **44**, 479 (1981).
 - ²⁹ J. L. Black and P. Fulde, Phys. Rev. Lett. **43**, 453 (1979).